Advanced (International) Macroeconomics

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Intertemporal Trade and Current Account

2-Period Model

2-period model

Within periods:
Inter- and intraindustry trade (see trade theory from Ricardo, Heckscher/Ohlin to new trade theories with imperfect markets). Gains from international labor division (comparative advantages), exploitation of economies of scale or intensified competition.

Across periods:
Intertemporal trade. Gains from international borrowing and lending.

Intertemporal choice - preferences

$$U = u(c_1) + \beta u(c_2), \quad 0 < \beta < 1 \tag{1}$$

- u period (instantaneous) utility
- $u' > 0, u'' < 0, \lim_{c \to 0} u'(c) = \infty$
- β subjective discount factor (time-preference parameter)

Marginal rate of substitution

$$MRS \left(\equiv -\frac{dc_2}{dc_1}\bigg|_{U=const} \right) = \frac{u'(c_1)}{\beta u'(c_2)} \tag{2}$$

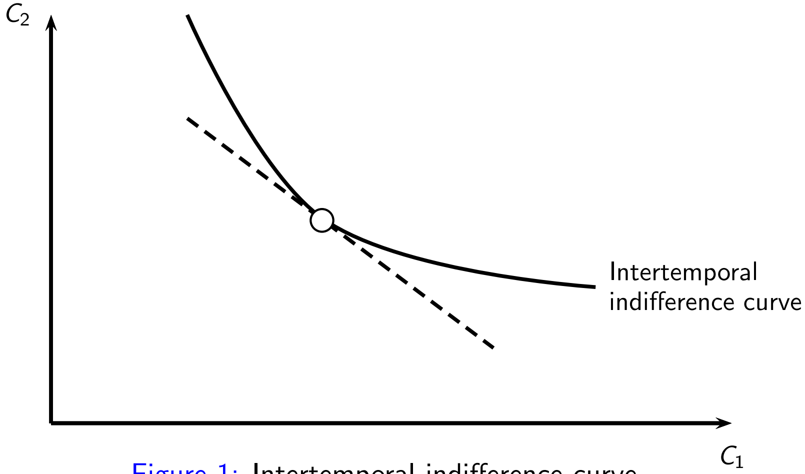


Figure 1: Intertemporal indifference curve

Utility functions and intertemporal budget constraint

Isoelastic utility functions

- Example 1: $u(c_t) = \ln c_t$
- Example 2: $u(c_t) = \dfrac{c_t^{1-1/\sigma}}{1-1/\sigma} \quad , \qquad 0 < \sigma (\neq 1).$

Intertemporal budget constraint

$$c_1 + \frac{c_2}{1+r} = y_1 + \frac{y_2}{1+r} \tag{3}$$

- r interest rate (if there are variable interest rates, r_{t+1} denotes interest rate from t to $t+1$. Then $\dfrac{c_2}{1+r_2}$ etc.).
- y_t endowment (income) in period t.

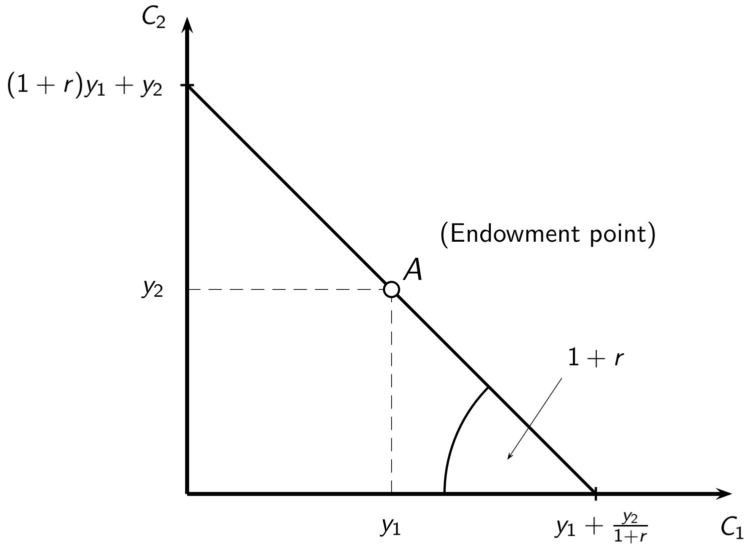


Figure 2: Intertemporal budget constraint

Optimal intertemporal choice

$$\max \quad U \qquad \text{s.t.} \quad (3)$$

Lagrange-Function:

$$\mathcal{L} = u(c_1) + \beta u(c_2) + \lambda \left(y_1 + \frac{y_2}{1+r} - c_1 - \frac{c_2}{1+r} \right)$$

Optimal intertemporal choice

First-order conditions

$$
\begin{aligned}
\frac{\partial \mathcal{L}}{\partial c_1} &= 0 \implies u'(c_1) = \lambda \\
\frac{\partial \mathcal{L}}{\partial c_2} &= 0 \implies \beta u'(c_2) = \frac{\lambda}{1+r} \\
\frac{\partial \mathcal{L}}{\partial \lambda} &= 0 \implies (3)
\end{aligned}
$$

combine to the so-called *intertemporal Euler equation*:

$$u'(c_1) = (1+r)\beta u'(c_2) \tag{4}$$

Eq. (4) determines how consumption needs to be allocated intertemporally in order to maximize utility at a given interest rate r.

Interpretation of Euler Equation

The intertemporal choice is optimal if there are no gains from intertemporal reallocation. Equation (4) is equivalent to

$$MRS = 1 + r \tag{5}$$

where MRS is given by (2)

Equilibrium in an endowment economy

Equilibrium in an endowment economy

Endowment economies have no capital accumulation and no production.

Aggregate supply (with symmetric agents):
$Y_t = y_t N_t,\ t = 1, 2$ where N_t is the population size in period t. In the following, N_t is normalized to 1 so that $Y_t = y_t$.

Aggregate demand:
$C_t = c_t,\ t = 1, 2$

Equilibrium in an endowment economy

In **closed economy** (autarky):

$$C_t = Y_t, \ t = 1, 2 \tag{6}$$

In **open economy**:

$$C_t = Y_t + r_t B_t - CA_t \tag{7}$$

where B_t is the value of *net foreign assets* inherited from period $t - 1$ and CA_t is the *current account balance* (Ertragsbilanz, auch Leistungsbilanz).

By definition,

$$CA_t = B_{t+1} - B_t \tag{8}$$

Remarks on national accounting

- *Gross domestic product* (GDP) (Bruttoinlandsprodukt): Y_t
- *Gross national product* (GNP) (Bruttosozialprodukt): $Y_t + r_t B_t$ i.e.
  - GNP=GDP+net international factor payments
  - net international factor payments here only includes interest and dividend earnings on net foreign assets
  - but no workers remittances
- *Trade balance* (goods and services): Net exports NX_t

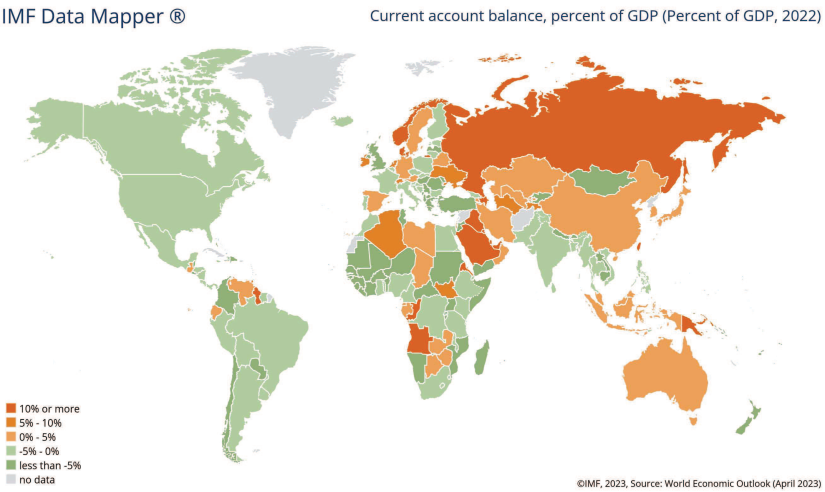
Remarks on national accounting

- *Capital account balance* (Kapitalverkehrsbilanz, auch Kapitalbilanz; includes financial account balance): Net sales of foreign assets: $-(B_{t+1} - B_t)$
- *Balance of payments* (Zahlungsbilanz): $NX_t + r_t B_t = B_{t+1} - B_t$
- *Current account balance* (Ertragsbilanz, auch Leistungsbilanz):

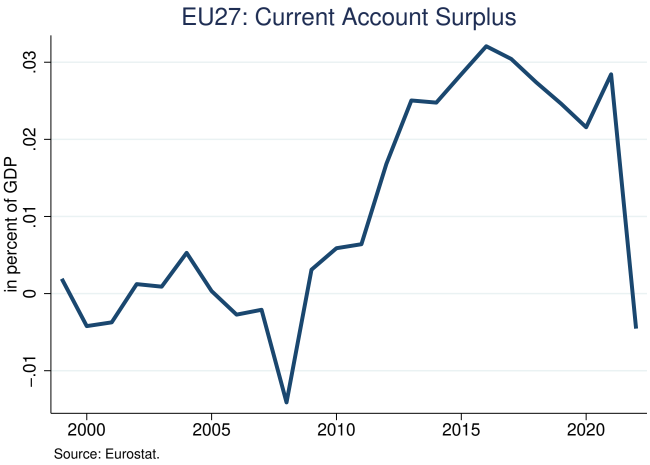
$$\text{within period perspective: } CA_t = NX_t + r_t B_t \tag{9}$$

$$\text{intertemporal perspective: } CA_t = B_{t+1} - B_t \tag{10}$$

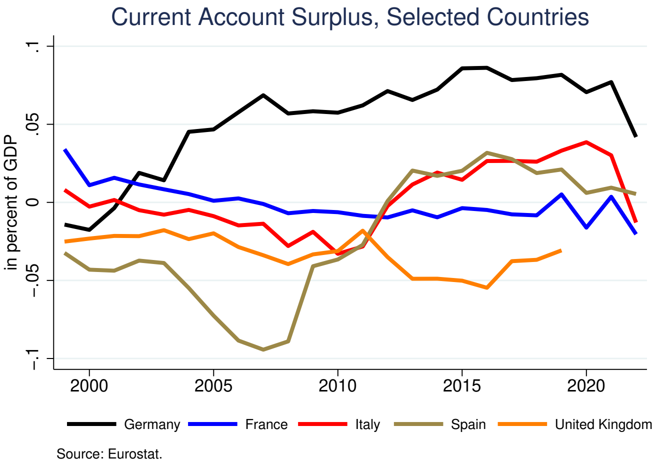
Remarks on national accounting



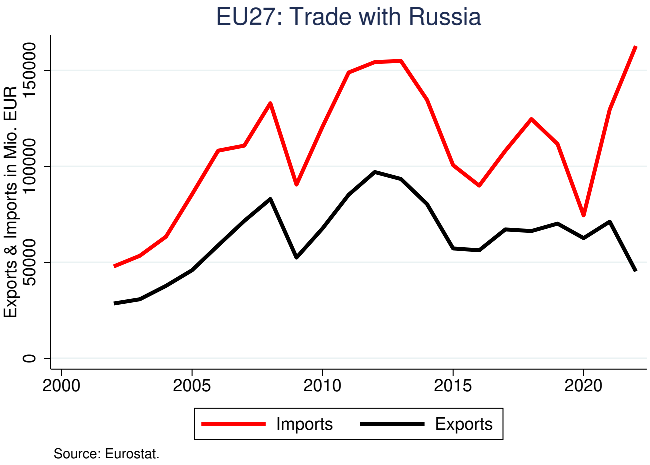
Remarks on national accounting



Remarks on national accounting



Remarks on national accounting



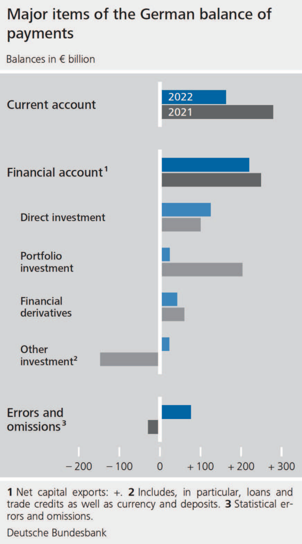
Remarks on national accounting

Major items of the balance of payments (€ billions)

Item	2020	2021	2022
I. Current account			
Total	+240.2	+278.7	+162.3
1. Goods[1]	+191.0	+194.4	+111.9
2. Services	+7.4	+4.8	–30.8
3. Primary income	+96.0	+138.5	+150.0
4. Secondary income	–54.2	–59.0	–68.8
II. Capital account			
Total	-9.1	-1.2	-18.6
III. Financial account balance[2]			
Total	+191.5	+248.6	+219.8
1. Direct investment	–4.9	+100.4	+125.3
2. Portfolio investment	+16.4	+203.5	+24.3
3. Financial derivatives[3]	+94.6	+60.2	+42.7
4. Other investment[4]	+85.4	–147.4	+23.1
5. Reserve assets	–0.1	+31.9	+4.4
IV. Errors and omissions			
Total	–39.6	–29.0	+76.2

Source: Deutsche Bundesbank: Monthly Report, March 2023. [1] Mainly foreign trade. [2] Increase in net external position: + / decrease in net external position: -. [3] Arising from options and financial futures contracts or employee stock options. [4] Includes loans and trade credits as well as currency and deposits.

Remarks on national accounting

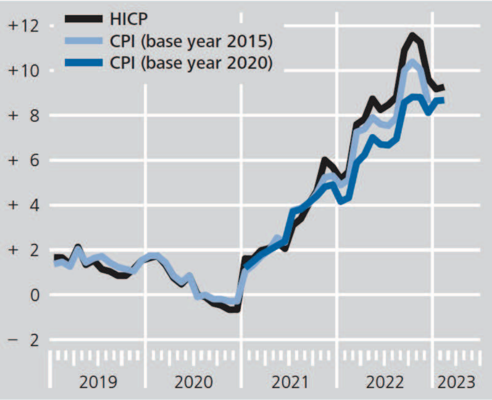


Source: Deutsche Bundesbank: Monthly Reports, March 2023

Development of consumer prices

Consumer prices in Germany: HICP and revised CPI

Year-on-year percentage change



Source: Federal Statistical Office.
Deutsche Bundesbank

Remarks on national accounting

Long-run impact of short-run imbalances:

$$CA_t = NX_t + r_t B_t \text{ and } CA_t = B_{t+1} - B_t$$

imply

$$B_{t+1} = NX_t + (1 + r_t)B_t$$

Repeating the argument for B_{t+2} we get

$$B_{t+2} = NX_{t+1} + (1 + r_{t+1})NX_t + (1 + r_t)(1 + r_{t+1})B_t$$

Remarks on national accounting

In a $T+1$-period world with $r_t = r$ (inheriting B_t and leaving B_{t+T+1}):

$$
\begin{aligned}
B_{t+T+1} &= (1+r)^{T+1} B_t + (1+r)^T NX_t + \ldots \\
&\quad + (1+r) NX_{t+T-1} + NX_{t+T}
\end{aligned}
$$

$$\Longleftrightarrow$$

$$\left(\frac{1}{1+r}\right)^T B_{t+T+1} = (1+r) B_t + \sum_{s=t}^{t+T} \left(\frac{1}{1+r}\right)^{s-t} NX_s \quad (11)$$

Remarks on national accounting

Terminal condition

$$B_{t+T+1} = 0$$

implies

$$\sum_{s=t}^{t+T} \left(\frac{1}{1+r}\right)^{s-t} NX_s = -(1+r)B_t$$

Remarks on national accounting

For instance, for $B_3 = B_1 = 0$ (temporary imbalance[1]):

$$\begin{aligned} NX_1 + \frac{NX_2}{1+r} &= -(1+r)B_1 = 0 \\ CA_1 + CA_2 &= 0 \end{aligned}$$

and

$$\begin{aligned} CA_1 &= B_2 - B_1 = B_2 \\ CA_2 &= B_3 - B_2 = -B_2 \\ NX_1 &= CA_1 - rB_1 = CA_1 = B_2 \\ NX_2 &= CA_2 - rB_2 = -(1+r)B_2 \end{aligned}$$

[1] if interested in more long-run dynamics, see Obstfeld/Rogoff Chapter 2

Equilibrium in a closed economy

Equilibrium in a closed economy

Goods market equilibrium

$$C_t = Y_t, \ t = 1, 2$$

and optimal consumption choice (cf. intertemporal Euler equation)

$$u'(C_1) = (1 + r)\beta u'(C_2)$$

give us *the autarky real interest rate*

$$1 + r^A = \frac{u'(Y_1)}{\beta u'(Y_2)} \tag{12}$$

Equilibrium in a closed economy

(Budget constraint (3) is obviously fulfilled for $C_t = Y_t$.)

- $1 + r^A$ is the willingness to pay for present consumption.
- Virtual price in closed economy without investment possibilities.
- Relevant when opening up.

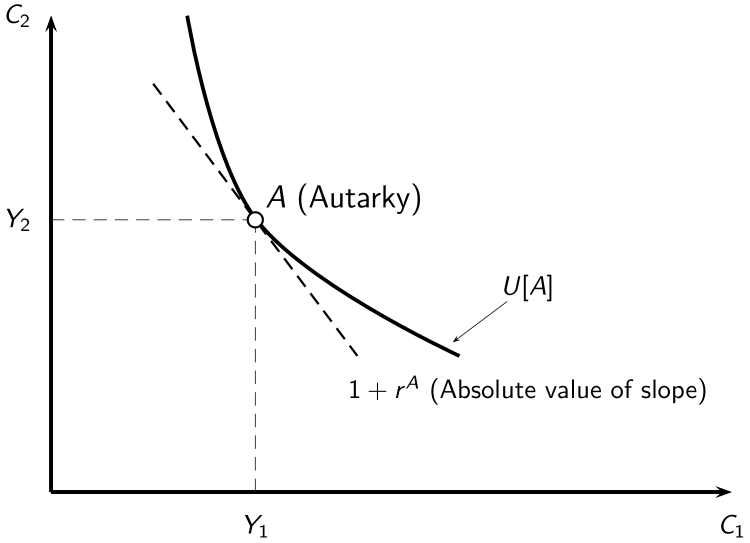


Figure 3: Autarky equilibrium

Effect of time preference on r^A

$\beta > \widetilde{\beta}$ implies

$$MRS(Y_1, Y_2) \quad < \quad \widetilde{MRS}(Y_1, Y_2)$$

$$\text{where } MRS(Y_1, Y_2) \quad \equiv \quad \frac{u'(Y_1)}{\beta u'(Y_2)}$$

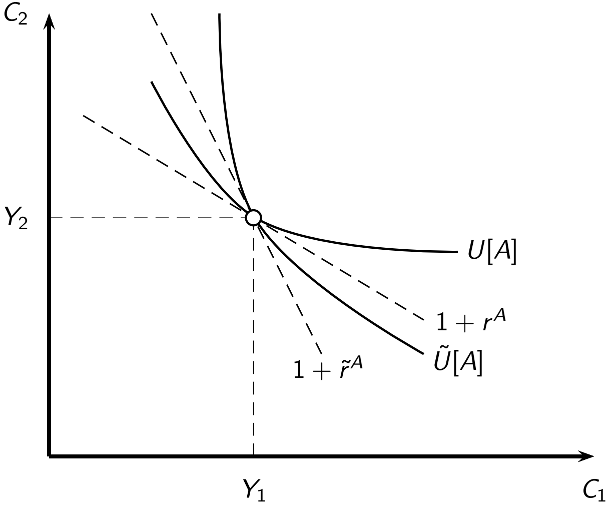


Figure 4: r^A rises with impatience.

Effect of output changes on r^A

Assume linear consumption expansion path $(MRS(\lambda Y_1, \lambda Y_2) = MRS(Y_1, Y_2))$

- r^A rises if positive output shock is expected.
- No change of r^A if present and future output rise pari passu.

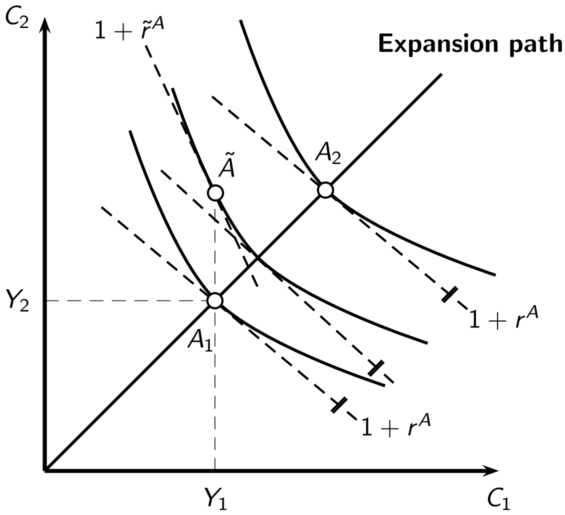


Figure 5: Effect of output changes on r^A

Equilibrium in a small open economy

Equilibrium in a small open economy

- 2 periods: $B_1 = B_3 = 0$ i.e. $NX_1 + \frac{NX_2}{1+r} = 0$
- r exogenously given by the world market

Intertemporal equilibrium allocation C_1, C_2 determined by:

$$u'(C_1) = (1+r)\beta u'(C_2) \tag{13}$$

$$C_1 + \frac{C_2}{1+r} = Y_1 + \frac{Y_2}{1+r} \tag{14}$$

Special case

If subjective discount factor is equal to market discount factor

$$\beta = \frac{1}{1+r},$$

the solution of (13) & (14) is given by

$$C_1 = C_2 \equiv \overline{C} \tag{15}$$

$$\overline{C} = \frac{(1+r)Y_1 + Y_2}{2+r} \tag{16}$$

For $\beta < \dfrac{1}{1+r}$ the allocation is biased in favor of C_1.

Open vs. closed economy equilibrium

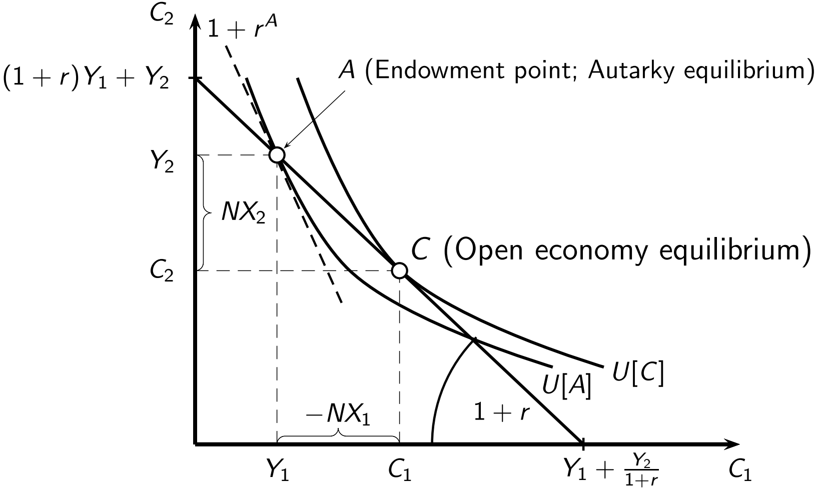


Figure 6: Comparing open and closed equilibrium if $r_A > r$

Open vs. closed economy equilibrium

- Access to international capital markets allows intertemporal income shifting.
- Here from future to present (borrowing) since $r < r^A$.
- Gains from intertemporal trade $U[C] > U[A]$
- Debt from current net imports $NX_1 = Y_1 - C_1 < 0$ must be paid back by future net exports
  $NX_2 = -(1+r)NX_1 = Y_2 - C_2 > 0$

Implications for trade flows and capital account

According to (7), (9) and (10):

$$\begin{aligned} C_1 = \quad Y_1 + rB_1 - CA_1 &= Y_1 + (1+r)B_1 - B_2 \\ &= Y_1 - NX_1 \end{aligned}$$

$$\begin{aligned} C_2 = \quad Y_2 + rB_2 - CA_2 &= Y_2 + (1+r)B_2 - B_3 \\ &= Y_2 - NX_2 \end{aligned}$$

In 2-period world: $B_1 = B_3 = 0$

Trade flows

$$\begin{aligned} C_1 - Y_1 &= -NX_1 \\ Y_2 - C_2 &= (1+r)(C_1 - Y_1) = NX_2 \end{aligned}$$

Net foreign assets

$$\begin{aligned} B_2 &= -(C_1 - Y_1) \\ Y_2 - C_2 &= -(1+r) B_2 = (1+r)(C_1 - Y_1) \end{aligned}$$

"Long-run" effects of short-run trade deficit

If endowment expectations are wrong, the associated short run trade deficit may have long-run implications (3 or more periods, $B_1 = 0$).

- $CA_1 = NX_1 = B_2$
- $CA_2 = rB_2 = rNX_1$
- $B_3 = CA_2 + B_2 = (1 + r)NX_1$

$B_{t+1} = B_t,\ t \geqslant 3$ requires $CA_t = NX_t + rB_t = 0$

$$\Longrightarrow \text{For } t \geqslant 3\text{: } NX_t = -r(1 + r)NX_1 \Longrightarrow C_{t(\geqslant 3)} < Y_t$$

$B_4 = 0$ requires $CA_3 = -B_3$

$$\Longrightarrow NX_3 = -(1 + r)^2 NX_1 \Longrightarrow \widetilde{C}_3 < C_{t(\geqslant 3)} < Y_3$$

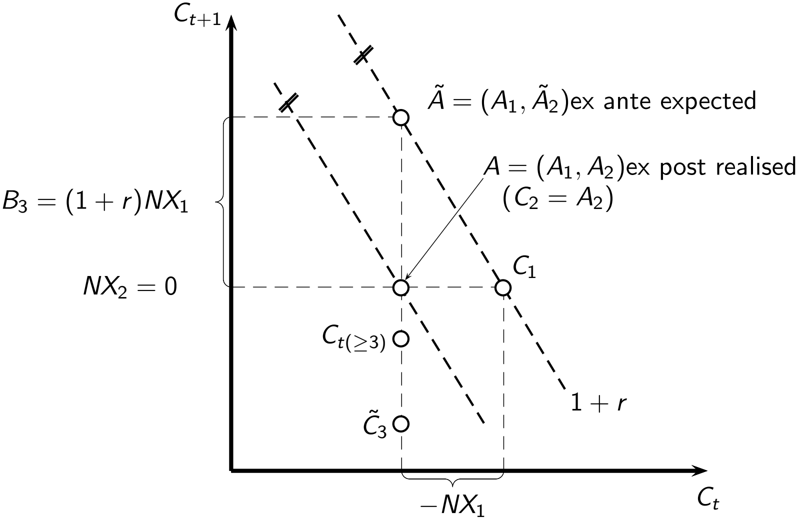


Figure 7: "Long-run" effects of trade deficit (3 per., $B_1 = 0$)

Intertemporal trade, savings and international equilibrium

Intertemporal trade, savings and international equilibrium

Case $r < r^A$

(see figure 6): Implies $NX_1 < 0$, $NX_2 > 0$, given $B_1 = B_3 = 0$.

- The country is a net importer in period 1, net exporter in period 2.
- $1 + r$ is the price of present consumption (here the import good) in terms of future consumption (export good).

Intertemporal trade, savings and international equilibrium

Terms of trade

$$\text{Terms of trade} \quad = \quad \frac{\text{price of exports}}{\text{price of imports}} = \frac{1}{1+r}$$

decline in r $\Rightarrow$ i) terms of trade improve

$\Rightarrow$ positive income and wealth effect on C_1

ii) substitution effects also favors C_1

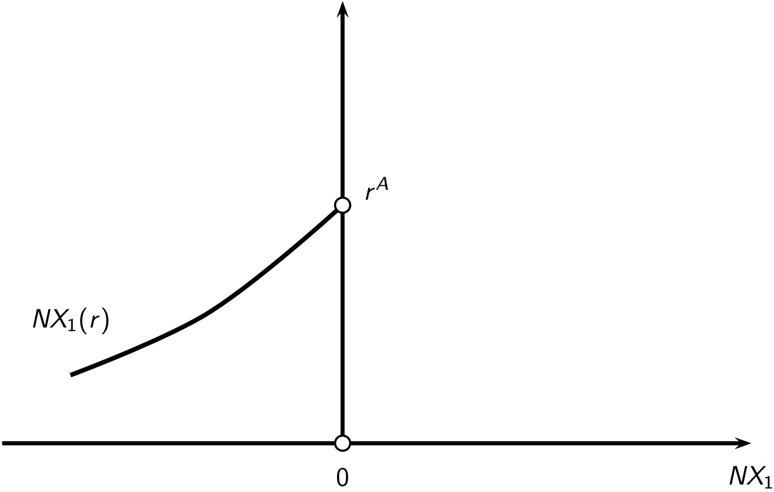


Figure 8: Net exports and interest rate if $r < r^A$

Intertemporal trade, savings and international equilibrium

Case $r > r^A$
(see figure 9): Implies $NX_1 > 0$, $NX_2 < 0$, given $B_1 = B_3 = 0$.

- Country is net exporter of present output, terms of trade $1 + r$

$$\begin{aligned} \textit{rise in } r \quad &\Rightarrow \quad \text{i) positive terms of trade effect on } C_1 \\ &\Rightarrow \quad \text{ii) negative substitution effect on } C_1 \end{aligned}$$

In sum, the NX_1-reaction is ambiguous.

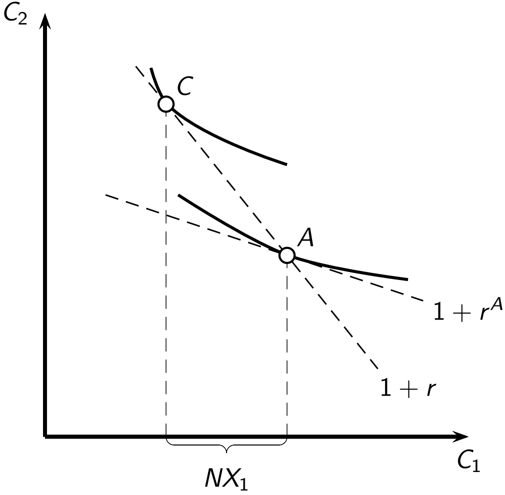


Figure 9: Intertemporal trade with $r > r^A$

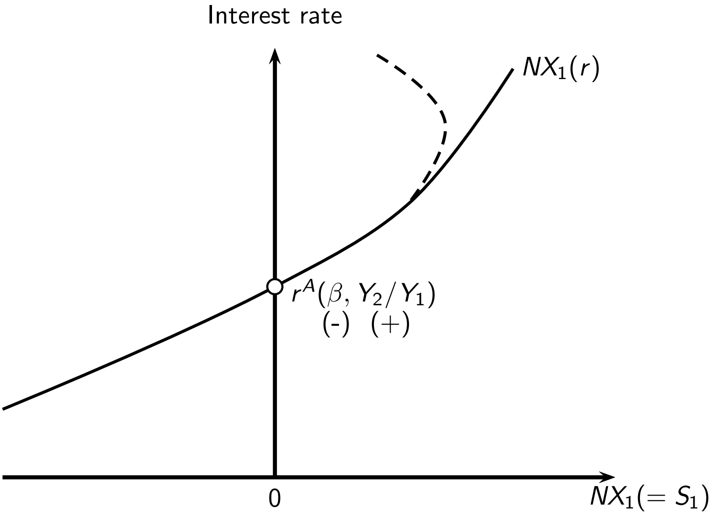


Figure 10: Net exports and interest rate if $r > r^A$

Intertemporal trade, savings and international equilibrium

The position of the curve is fixed by r^A.

Remember: r^A declines in β and (for linear expansion path of consumption) rises with Y_2/Y_1.

Moreover: For $B_1 = 0$, $CA_1 = NX_1$ and thus
$S_1 \equiv Y_1 + rB_1 - C_1 = NX_1$

International equilibrium – 2 large economies

Two country world:

$$\begin{aligned} \text{Home:} \quad & r^{A}\left(\beta, Y_2/Y_1\right) \\ \text{Foreign:} \quad & r^{A^*}\left(\beta^*, Y_2^*/Y_1^*\right) \end{aligned}$$

integrated world is like closed economy with goods market equilibrium condition

$$C_t + C_t^* = Y_t + Y_t^*$$

Using $C_t + NX_t = Y_t,\ C_t^* + NX_t^* = Y_t^*$, we get

$$NX_t + NX_t^* = 0 \tag{17}$$

This determines world interest rate r.

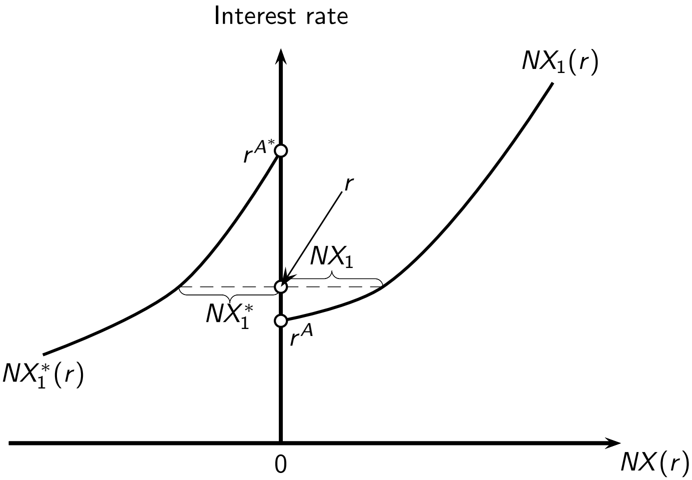


Figure 11: Equilibrium world interest rate if $r^{A^*} > r^A$

International equilibrium – 2 large economies

If increasing impatience (β or $\beta^* \downarrow$) or rising future output (Y_2/Y_1 or $Y_2^*/Y_1^* \uparrow$) raise r^A or r^{A^*} the equilibrium world interest rate rises, ceteris paribus. This

- worsens terms of trade for net importer Foreign
- improves terms of trade for net exporter Home

Capital accumulation and production

Capital accumulation and production – Assumptions

Production function:

$$Y_t = F(K_t)$$

Properties:

- $F(0) = 0$
- $F' > 0$
- $F'' < 0$

Inada conditions:

- $\lim_{K \to 0} F'(K) = \infty$
- $\lim_{K \to \infty} F'(K) = 0$

Since $N_t = 1$, level of capital stock K_t and capital intensity $k_t = K_t/N_t$ coincide.

Capital accumulation and production – Assumptions

Capital accumulation:

$$K_{t+1} = K_t + I_t \tag{18}$$

(Depreciation ignored, K can be eaten up, i.e. $I_t = -K_t$.)

Capital demand under perfect competition:

$$r_t = F'(K_t) \tag{19}$$

Wages (labor demand) under perfect competition:

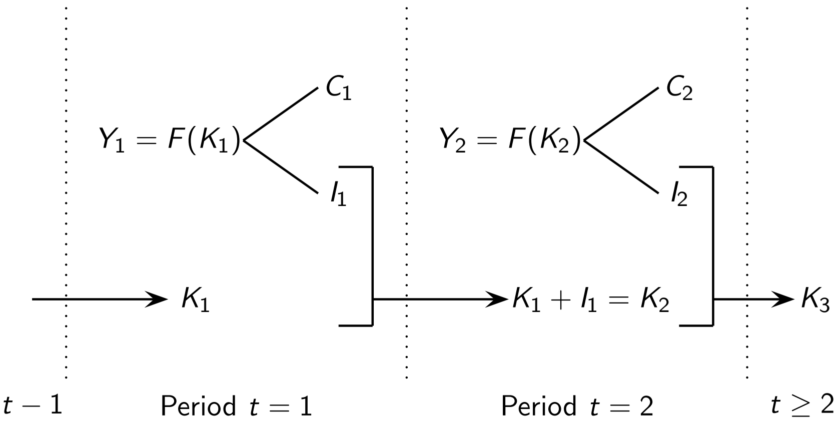
$$W_t = F(K_t) - r_t K_t \tag{20}$$

Closed economy with capital accumulation and production

Intertemporal production and investment – Autarky

Goods market equilibrium

$$C_t + I_t = Y_t \tag{21}$$



Intertemporal transformation curve ("Production possibilities Frontier" PPF)

$$\begin{aligned} C_2 + K_3 &= F(K_2) + K_2 \\ &= F\left[K_1 + \underbrace{F(K_1) - C_1}_{I_1}\right] + K_1 + F(K_1) - C_1 \quad (22) \end{aligned}$$

Intertemporal PPF:

$$C_2^+ \equiv C_2 + K_3$$

$$C_2^+ = F\left(\underbrace{K_1 + F(K_1) - C_1}_{K_2}\right) + \underbrace{K_1 + F(K_1) - C_1}_{K_2}$$

Intertemporal transformation curve ("Production possibilities Frontier" PPF)

$$\frac{dC_2^+}{dC_1} = -\left[1 + F'\left(\underbrace{K_1 + F(K_1) - C_1}_{K_2}\right)\right] < 0$$

$$\frac{d^2 C_2^+}{dC_1^2} = F''(K_2) < 0$$

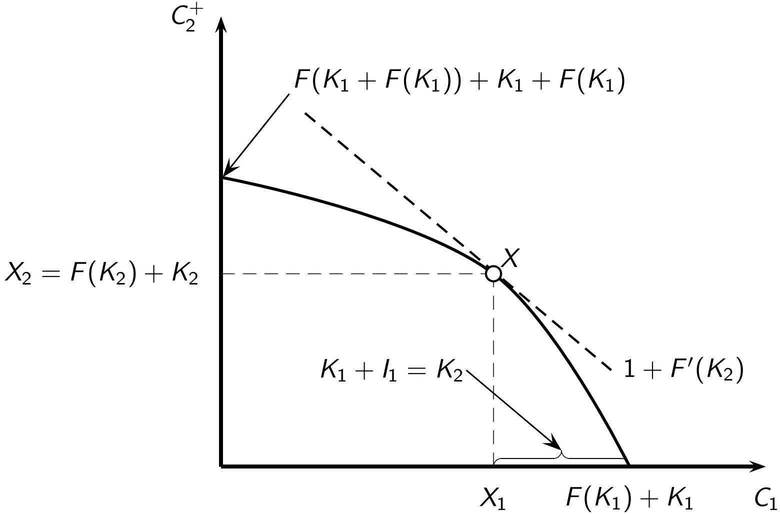
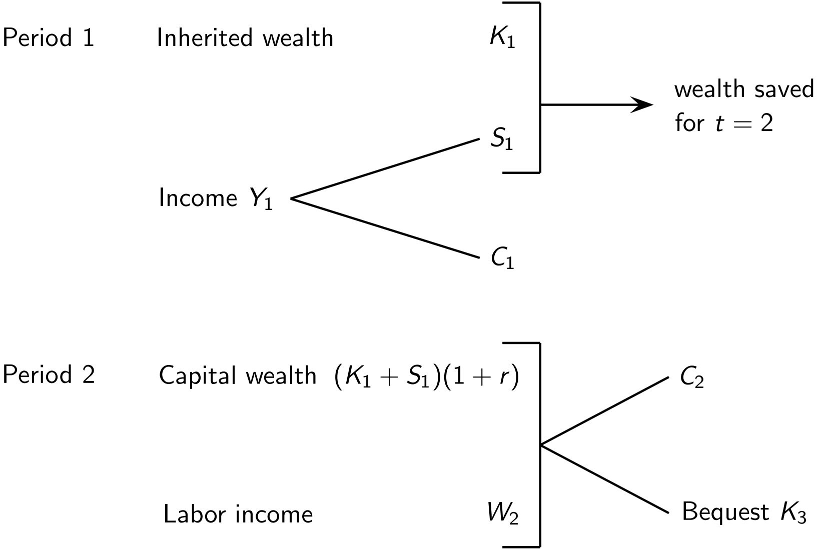


Figure 12: Intertemporal production possibilities frontier

Intertemporal budget constraint of representative HH



Intertemporal consumption possibility line (CPL)

Intertemporal budget constraint:

$$C_2 + K_3 = W_2 + (K_1 + Y_1 - C_1)(1 + r) \tag{23}$$

Since $W_2 = F(K_2) - rK_2$ and $K_1 + Y_1 - C_1 = K_2$ (23) is consistent with (22).

That means: Savings behavior of households leads to a point on the economy's PPF.

The question is: which point?

Optimal intertemporal choice

K_3- choice depends on the "bequest" motive. Can be captured by

$$u(C_1) + \beta u(C_2^+)$$

$$\begin{aligned} C_2^+ &= C_2 + K_3 \ldots \text{ "bequest" motive} \\ C_2^+ &= C_2 \ldots \text{ no "bequest" motive} \end{aligned}$$

$$\max\ u(C_1) + \beta u(C_2^+) \text{ s.t. } C_2^+ = W_2 + (K_1 + Y_1 - C_1)(1 + r)$$

Optimal intertemporal choice

Optimal intertemporal choice yields first-order condition

$$MRS \equiv \frac{u'(C_1)}{\beta u'(C_2^+)} = 1 + r$$

where $K_3 = 0$ without bequest motive. ($K_3 = 0$ implies $K_2 + I_2 = 0$ and thus $S_2 = I_2 = -K_2$.) In dubio, assume $K_3 = 0$, i.e. $C_2 = C_2^+$ in the following.

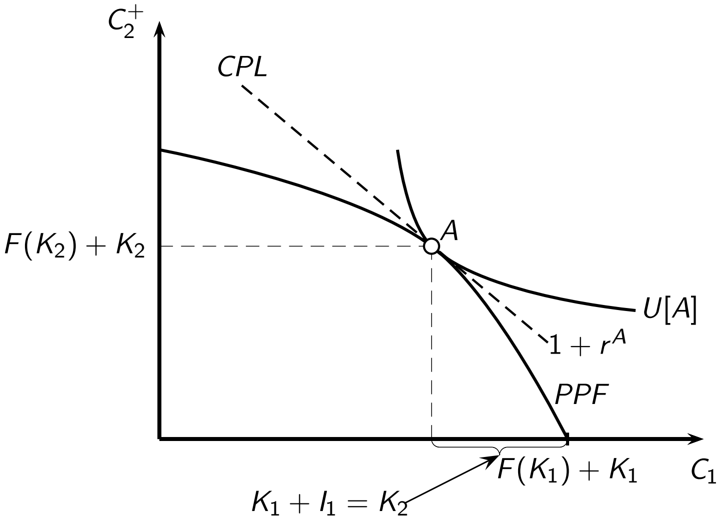


Figure 13: CPL and PPF

Small open economy with capital accumulation and production

SMOPEC with capital accumulation and production

Goods market equilibrium

$$C_t + I_t + NX_t = Y_t \tag{24}$$

and intertemporal foreign account (see (11))

$$NX_2 + (1+r)NX_1 = \underbrace{B_3 - (1+r)^2 B_1}_{D}$$

imply

$$\begin{aligned} C_2 + (1+r)C_1 &= Y_2 - I_2 + (1+r)(Y_1 - I_1) - D \\ &= F(K_2) - [K_3 - K_2] + (1+r)[F(K_1) - \\ &\quad (K_2 - K_1)] - D \end{aligned}$$

SMOPEC with capital accumulation and production

Hence,

$$\underbrace{C_2 + K_3}_{C_2^+} + (1+r)C_1 = \underbrace{F(K_2) + K_2}_{X_2} + (1+r)\underbrace{[F(K_1) + K_1 - K_2]}_{X_1} - D \tag{25}$$

where $X = (X_1, X_2)$ is a point at the PPF.

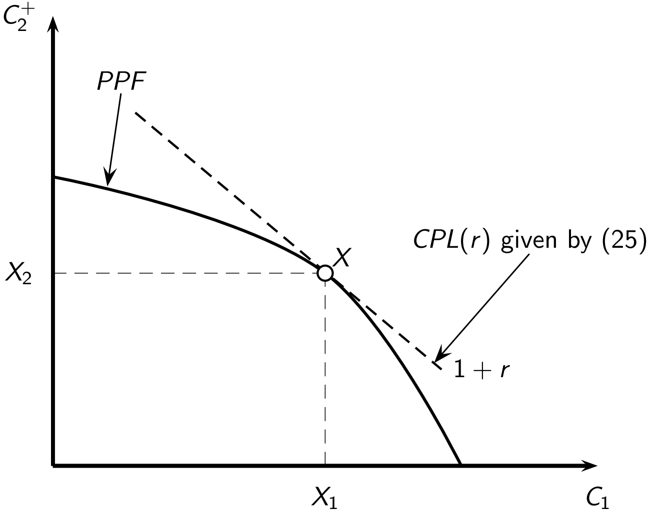


Figure 14: Consumption possibilities line (CPL) under world interest and $D = 0$.

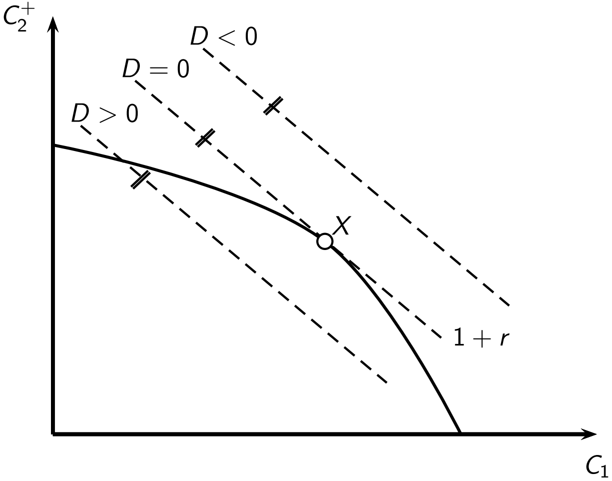


Figure 15: CPL under long-run imbalances ($D \neq 0$)

In the following $D = 0$ (e.g. $B_1 = B_3 = 0$).

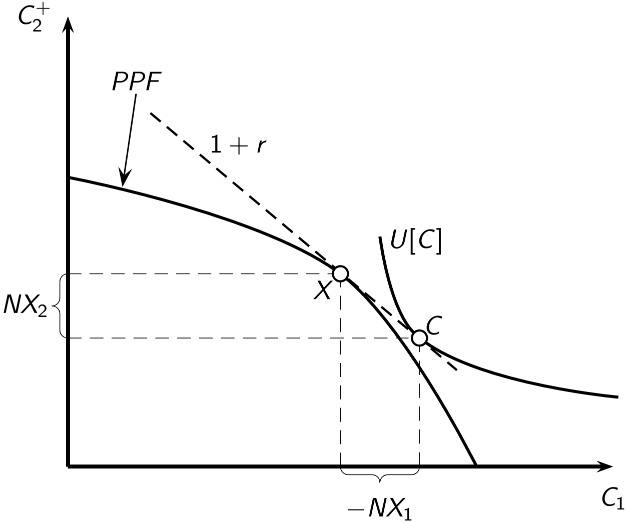


Figure 16: Equilibrium production (X) equilibrium consumption (C), and trade balances (NX)

From autarky to open economy equilibrium

Consider $r < r^A$:

In addition to the picture for the endowment economy: Production structure shifts from A to X by higher investments ΔI_1.

Increase in current consumption by ΔC_1. Current account deficit $-NX_1 = \Delta I_1 + \Delta C_1$ paid back by increased future production (+ possibly lower consumption).

Case $r < r^A$:

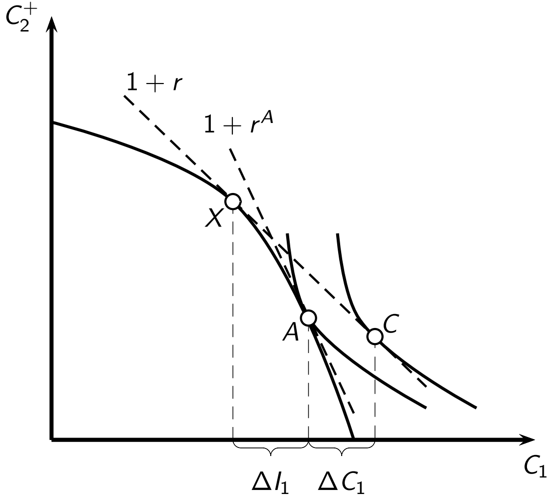


Figure 17: Double gains from intertemporal trade

Case $r > r^A$:

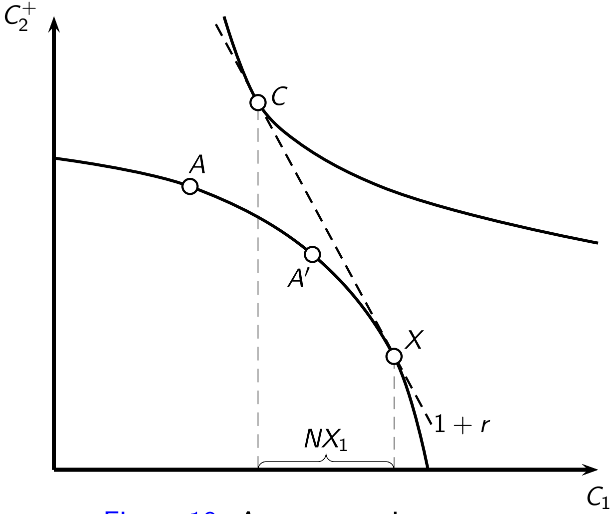


Figure 18: A net exporting country

From autarky to open economy equilibrium

Production shifts in favor of current output by decreasing investment $\Delta I_1 < 0$. Additional output allows net exports.

Net exports today allow higher future consumption $\Delta C_2 > 0$ by future imports ($NX_2 = -(1 + r)NX_1$). Present consumption C_1 may shrink (A') or increase (A) depending on the relative strength of income and substitution effect (plus output shift).

Adding government consumption

Adding government consumption

With government consumption, period utility has the following additive form: $u(C) + v(G)$. The budget constraint in the two period model is

$$C_1 + \frac{C_2}{1+r} = Y_1 - T_1 - I_1 + \frac{Y_2 - T_2 - I_2}{1+r},$$

where T_t denotes taxes and $Y_t - T_t$ is 'disposable' income of the private sector in period t.

Goods market equilibrium in period t:

$$C_t + I_t + G_t + NX_t = Y_t \tag{26}$$

Adding government consumption

Current account balance (recall (9),(10)):

$$\begin{aligned} CA_t &= NX_t + rB_t \\ &= Y_t + rB_t - C_t - G_t - I_t \\ &= \underbrace{Y_t + rB_t - C_t - T_t}_{S_t^P \text{ private savings}} + \underbrace{T_t - G_t}_{\text{public savings}} - I_t \end{aligned}$$

With a balanced budget $T_t = G_t$ of the public sector, private savings are equal to total savings ($S_t^P = S_t$) and

$$CA_t = \underbrace{S_t^P + T_t - G_t}_{S_t \text{ total savings}} - I_t \tag{27}$$

$$B_{t+1} = B_t + S_t - I_t \tag{28}$$

Impact of G in small open economy

Increase in $G_1(G_2)$ shifts transformation curve (PPF) for private sector leftward (downward).

In the following illustration (with a balanced budget of the government: $T_t = G_t$):

Initial situation:	$G_1 = G_2 = 0$ and $NX_1 = NX_2 = 0$
Shock 1:	$G_1 \uparrow$
Shock 2:	$G_2 \uparrow$

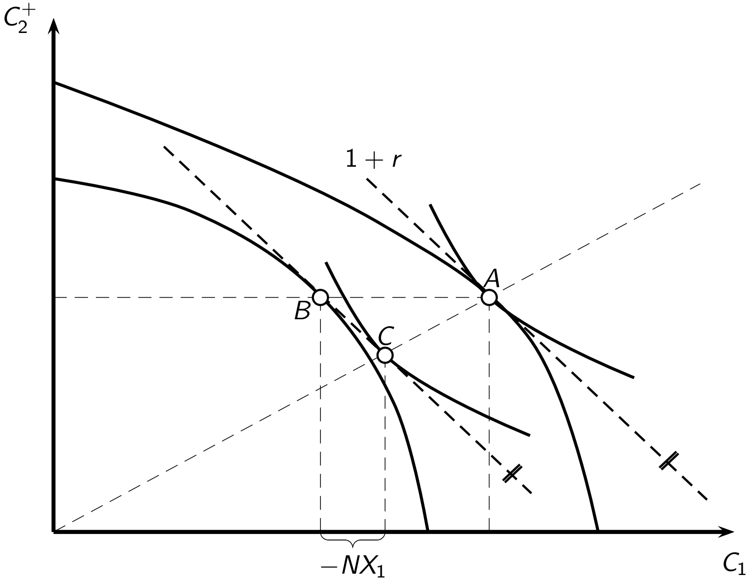


Figure 19: Impact of G_1 increase

Impact of G_1 increase

- Private feasible output shifts from A to B.
- Would decrease C_1 by the full amount of $G_1 = \overline{BA}$ leaving C_2 unaffected
- Individuals prefer C by borrowing from abroad.

Impact of anticipated G_2 increase on next slide

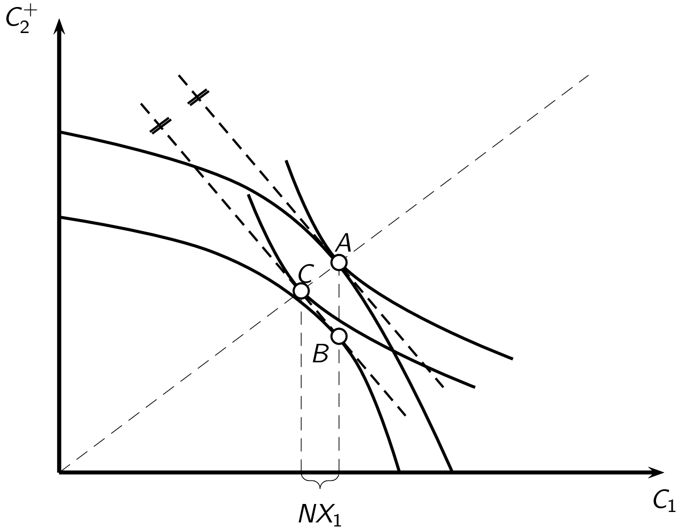


Figure 20: Impact of anticipated G_2 increase

Individuals “hedge” against tax G_2 by lending to *Foreign* in Period 1.

Investment, savings and world interest rate in international equilibrium

Investment, savings and world interest rate in international equilibrium

The investment function

Production function:

$$Y_t = A_t F(K_t)$$

A_t: Productivity parameter

Accumulation equation:

$$K_2 = K_1 + I_1$$

In the following, we consider a 2-period model with $B_1 = B_3 = 0$, $K_3 = 0$ and $G_t = T_t = 0$.

The investment function

Condition for optimal capital input under perfect competition:

$$r = A_2 F'(K_1 + I_1) \tag{29}$$

(29) defines investment curve

$$I_1 = I(r/A_2),\ I^{'} < 0$$

The negative slope follows from $F'' < 0$.

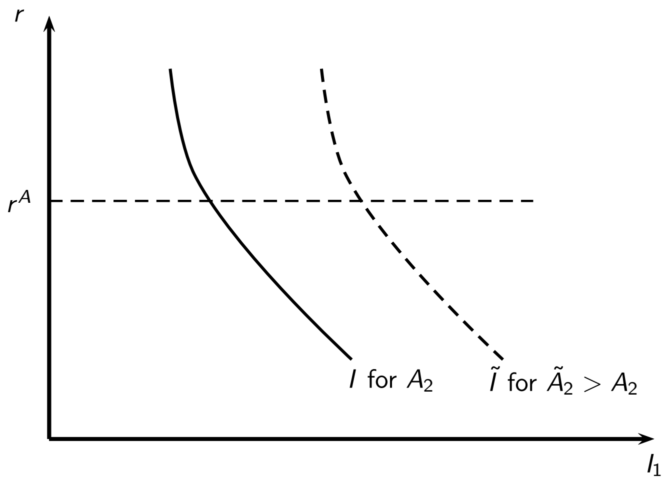


Figure 21: Investment curve and productivity shifts

Shifts in A_1 have no effect on investment since K_1 is already fixed from past decisions.

The saving function

Reconsidering the endowment economy: From the endowment economy we know that $B_1 = B_3 = 0$ implies $S_1 = Y_1 - C_1 = NX_1(r)$.

Furthermore, we can note that, $dS_1/dr = dNX_1/dr = -dC_1/dr$.

To determine the impact of interest rate r on savings (or, equivalently, NX_1), we can first look at the intertemporal Euler equation $u'(C_1) = (1+r)\beta u'(C_2)$.

Substituting the budget constraint $C_2 = (1+r)(Y_1 - C_1) + Y_2$ gives

$$u'(C_1) = (1+r)\beta u'\left((1+r)(Y_1 - C_1) + Y_2\right). \tag{30}$$

The saving function

Implicitly differentiating (30) with respect to r gives

$$\frac{dC_1}{dr} = \frac{\beta u'(C_2) + \beta(1+r)u''(C_2)(Y_1 - C_1)}{u''(C_1) + \beta(1+r)^2 u''(C_2)}. \tag{31}$$

Noting $u' > 0$, $u'' < 0$, it is immediate that $dNX_1(r)/dr = -dC_1/dr > 0$ if $C_1 > Y_1$ (or, equivalently, $NX_1 < 0$).

However, $dNX_1(r)/dr = -dC_1/dr < 0$ cannot be ruled out if $Y_1 > C_1$ (or, equivalently, $NX_1 > 0$) – see Figure 10.

The saving function

Consumption in a model with capital accumulation and production

Substituting the budget constraint

$$C_2 = (1+r)[A_1 F(K_1) - C_1 - I_1] + A_2 F(K_1 + I_1) + K_1 + I_1$$

for C_2 in the Euler equation $u'(C_1) = (1+r)\beta u'(C_2)$, gives

$$\begin{aligned} u'(C_1) = (1+r)\beta u' \{ & (1+r)[A_1 F(K_1) - C_1 - I_1] \\ & + A_2 F(K_1 + I_1) + K_1 + I_1 \} . \end{aligned} \tag{32}$$

Implicitly differentiating with respect to r yields

$$\begin{aligned} \frac{dC_1}{dr} = & \frac{\beta u'(C_2) + \beta(1+r)u''(C_2)\left[A_1 F(K_1) - C_1 - I_1\right]}{u''(C_1) + \beta(1+r)^2 u''(C_2)} \\ & + \frac{\beta(1+r)u''(C_2)\left\{A_2 F'(K_1 + I_1) - r\right\} \partial I / \partial r}{u''(C_1) + \beta(1+r)^2 u''(C_2)} . \end{aligned}$$

The saving function

Accounting for $A_2F'(K_1 + I_1) = r$ further implies

$$\frac{dC_1}{dr} = \frac{\beta u'(C_2) + \beta(1+r)u''(C_2)\left[A_1F(K_1) - C_1 - I_1\right]}{u''(C_1) + \beta(1+r)^2u''(C_2)}. \tag{33}$$

Hence, the derivative in (33) is precisely the same as the derivative in (31), but with $Y_1 - C_1$ replaced by the date 1 current account for an investment economy with $B_1 = 0$: $A_1F(K_1) - C_1 - I_1$.

That means that, given current account balances, the slope of the saving schedule is the same as for the endowment economy!

The saving function

<u>An intuition for this result</u>

The symmetry in the reaction of savings to interest rate adjustments in the endowment and the investment economy is a consequence of the *envelope theorem*.

The first-order condition for profit-maximizing investment ensures that a small deviation from optimum investment does not alter the present value of national output, evaluated at the world interest rate.

Consequently, at the margin, the investment adjustment $\partial I_1/\partial r$ has no effect on net lifetime resources, and hence no effect on consumption response.

The saving function

From consumption to saving

As noted above, savings in period 1 are given by $S_1 = Y_1 - C_1$ or, equivalently, $S_1 = A_1 F(K_1) - C_1$. Hence, we can write savings as function of r, A_1, A_2 and β:

$$S_1 = S(r, A_1, A_2, \beta),$$

with $\partial S_1 / \partial r > 0$ in the regular (*non-perverse*) case.

Saving curve and productivity shift

An increase of A_t has analogous effects to an increase of Y_t in endowment economy.

- According to slide 52 a rise in Y_1 shifts the S-curve to the right. A rise in Y_2 shifts the S-curve to the left.
- Rising impatience (a fall in β) also shifts the saving curve to the left.

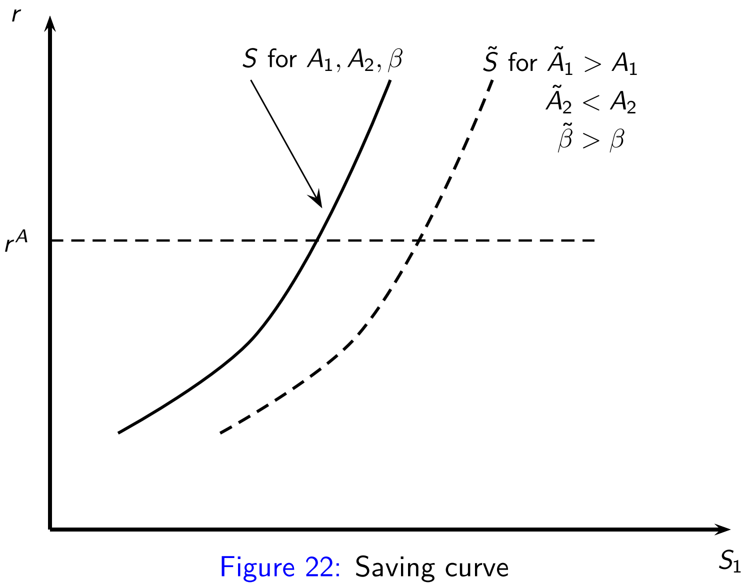


Figure 22: Saving curve

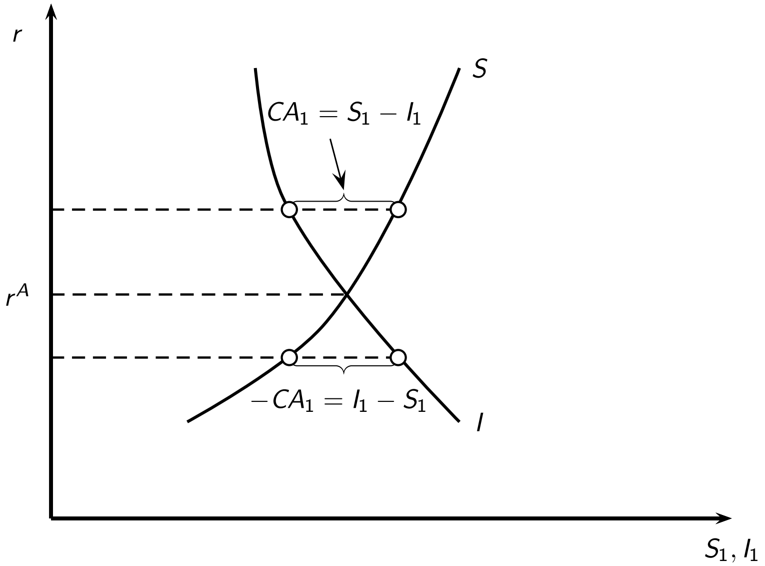


Figure 23: Investment, savings, and current account

International equilibrium in a two-region world

The Metzler diagrams

World equilibrium requires

$$CA_1 + CA_1^* = 0$$

i.e.

$$S_1 - I_1 = -\left(S_1^* - I_1^*\right)$$

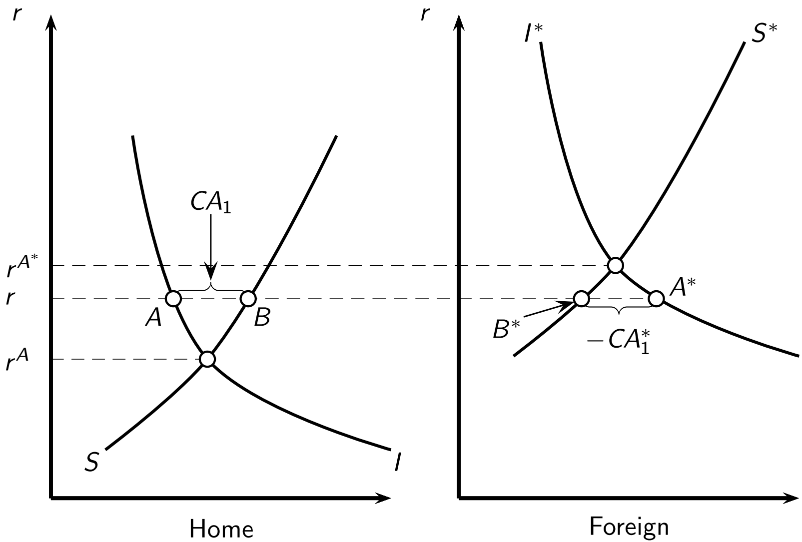


Figure 24: World equilibrium interest rate $r^A < r < r^{A*}$

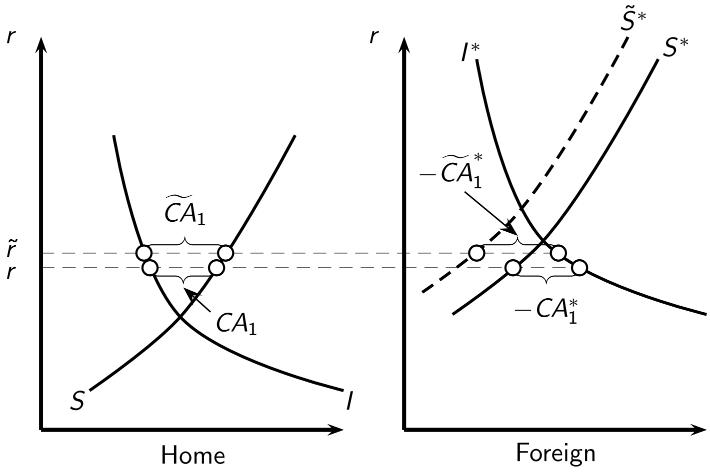


Figure 25: Impact of rising impatience in *Foreign* ($\beta^* \downarrow$)

World interest rate rises and current account CA_1 from *Home* to *Foreign* increases. Investment decreases in both regions.

Impact of positive productivity shock in *Foreign*

Consider a productivity shock of the form $A_2^* \uparrow$

World interest rate rises

In *Home*	Investment falls Saving and CA_1-surplus rise
In *Foreign*	Investment reaction ambiguous CA_1^*-deficit rises

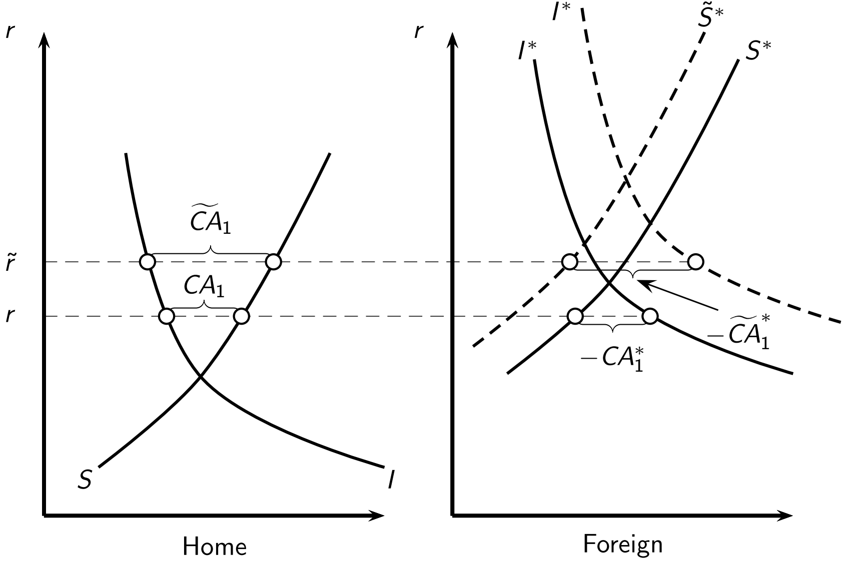


Figure 26: Impact of positive productivity shock in Foreign

Stability and Marshall-Lerner condition

The world equilibrium condition $CA_t + CA_t^* = 0$ is

$$S_t(r) + S_t^*(r) = I_t(r) + I_t^*(r) \tag{34}$$

(Use $CA_t = S_t - I_t$)

As addressed in Figure 10, the saving curve may be backward bending, so that multiple equilibria and unstable equilibria cannot be excluded.

Stability and Marshall-Lerner condition

(Walrasian) stability condition: A market is stable in the Walrasian sense if a small increase in the price of the good traded there causes excess supply, while a small decrease causes excess demand.

The stability condition defining Walrasian stability in the market for world savings is that a small rise in r should lead to an excess supply of savings:

$$\frac{d\left[S_1(r)+S_1^*(r)\right]}{dr} > \frac{d\left[I_1(r)+I_1^*(r)\right]}{dr} \tag{35}$$

Stability guarantees that market forces tend to eliminate imbalances resulting from small disturbances of international equilibrium.

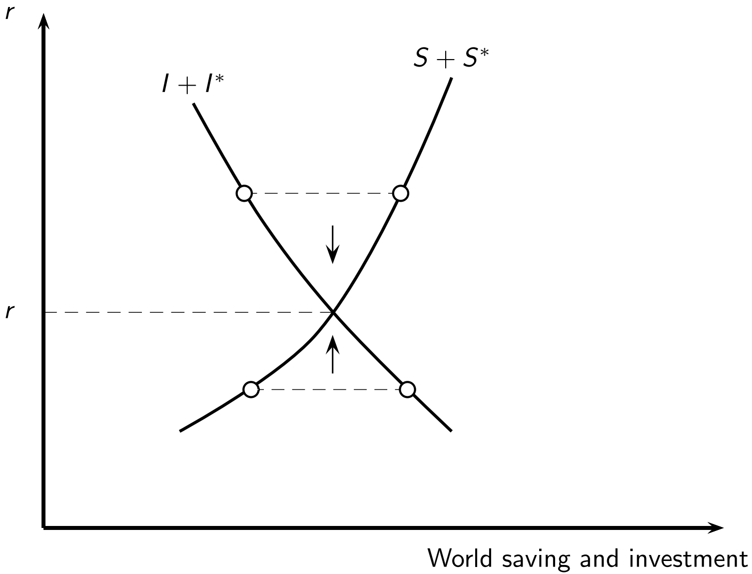


Figure 27: Savings and Investment

Stability and Marshall-Lerner condition

For $B_1 = B_3 = 0$ national accounting identities imply

$$\begin{aligned} NX_1 &= CA_1 = S_1 - I_1 \\ NX_1^* &= CA_1^* = S_1^* - I_1^* \end{aligned}$$

Moreover (see (11)),

$$NX_1^* + \frac{NX_2^*}{1+r} = 0$$

Stability and Marshall-Lerner condition

Using this in international equilibrium condition (34), we get

$$S_1 - I_1 + S_1^* - I_1^* = NX_1 - \frac{NX_2^*}{1+r}$$

Thus (35) is equivalent to

$$\frac{d\left[NX_1(r) - \frac{NX_2^*(r)}{1+r}\right]}{dr} > 0 \qquad (36)$$

Stability and Marshall-Lerner condition

$$
\begin{aligned}
\frac{d\left[NX_1(r) - \dfrac{NX_2^*(r)}{1+r}\right]}{dr} &= NX_1' - \frac{NX_2^{*'}(1+r) - NX_2^*}{(1+r)^2} \\
&= \frac{NX_2^*}{(1+r)^2}\left[\frac{(1+r)NX_1'}{NX_1}\frac{NX_1(1+r)}{NX_2^*}\right. \\
&\qquad \left. - \frac{NX_2^{*'}(1+r)}{NX_2^*} + 1\right]
\end{aligned}
$$

Stability and Marshall-Lerner condition

In equilibrium $NX_1(1+r) = -NX_2 = NX_2^*$. Thus the square bracket is negative (positive) if

$$\underbrace{-\frac{(1+r)NX_1'}{NX_1}}_{\eta} + \underbrace{\frac{(1+r)NX_2^{*\prime}}{NX_2^*}}_{\eta^*} > (<)\, 1 \tag{37}$$

If Home is net importer today ($NX_1 < 0$), then $NX_2^* < 0$ and stability condition (35) is equivalent to

$$\eta + \eta^* > 1. \tag{38}$$

Stability and Marshall-Lerner condition

Interpretation ($NX_1 < 0$)

η is the (absolute value of) negative import elasticity of Home with respect to price $1 + r$ of current consumption. η^* is the (positive) elasticity of Foreign's future imports. (38) is the intertemporal analogue to the so-called *Marshall-Lerner* condition.

Remark

When *Home* happens to be the exporter in period 1, rather than the importer, (38) still characterizes the Walras-stable case, but with import elasticities defined so that Home's and Foreign's role are interchanged.